the three following equations are available:

(1) The equation of equilibrium in the radial direction:

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_t}{r}=0 \quad . \quad . \quad . \quad (12)$$

(2) The condition of compatibility:

$$r\frac{d\epsilon_t}{dr}=\epsilon_r-\epsilon_t \quad . \quad . \quad . \quad . \quad (13)$$

(3) The condition which expresses that a plane section perpendicular to the axis remains plane after deformation:

$$\epsilon_{a_i}=\epsilon_{a_o} \quad . \quad . \quad . \quad . \quad (14)$$

where ϵ_{a_o} is the strain parallel to the axis at the outer radius of the cylinder.

Using the system composed of equations (12), (13), and (14) it is possible to derive σ_{r_i}, σ_{t_i}, and σ_{a_i} from $\sigma_{r_{i-1}}$, $\sigma_{t_{i-1}}$, and $\sigma_{a_{i-1}}$. The system is solved by a method of continual approach whose layout is described below.

Starting point of the calculations

The calculations are started at the outer radius of the cylinder, where the boundary conditions impose $\sigma_{r_o}=0$. The two other stresses at the outer radius are selected arbitrarily: σ_{t_o} and σ_{a_o}. The stress field at any point in the cylinder wall can then be calculated by progressing inwards. The boundary condition at the inner radius gives the pressure, p:

$$p=-\sigma_{r_n} \quad . \quad . \quad . \quad . \quad (15)$$

The calculation is thus carried out 'backwards', i.e. the stress distribution is not calculated starting from the internal pressure; on the contrary, the starting point is the stress field at the outer radius, r, and the internal pressure that has to be applied to the cylinder to create this stress field is derived from this starting point.

In actual fact, the problem is not so simple, because σ_{t_o} and σ_{a_o} are not independent variables. If one of these two stresses is chosen arbitrarily, the choice of the other is no longer free. To remove this difficulty use is made of the fact that the internal pressure can be derived, not only from equation (15) but also by equating the force acting on the heads due to the internal pressure with the sum of the axial stresses in a section normal to the axis. The p_a value derived by this means is:

$$p_a=\frac{2}{r_n^2}\int_{r_n}^{r_o}\sigma_a r\,dr \quad . \quad . \quad . \quad (16)$$

The p and p_a values derived respectively from equations (15) and (16) must be equal.

The calculation will thus be carried out by fixing the value of σ_{t_o} and assuming, as a first approximation, that σ_{a_o} has a value $\sigma_{a_o}{}^{\mathrm{I}}$, as shown in Fig. 8 which gives the general layout of the calculations. In general, the p and p_a values will not be equal: by comparing these two values, a second approximation of σ_{a_o}, i.e. $\sigma_{a_o}{}^{\mathrm{II}}$, can be calculated, the value of σ_{t_o} remaining the same. The calculation is then continued until identical values are obtained for p and p_a.

Calculation of the stresses at the inner radius of a given zone

The stresses at the inner radius of zone i are calculated starting from a first approximation ($\sigma_{r_i}{}^{\mathrm{I}}$, $\sigma_{t_i}{}^{\mathrm{I}}$, $\sigma_{a_i}{}^{\mathrm{I}}$) obtained

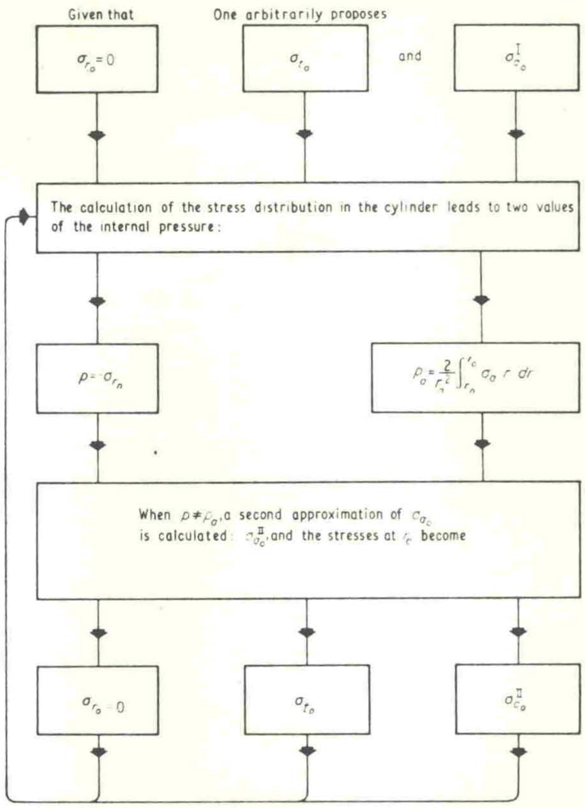

Fig. 8. General layout of calculations

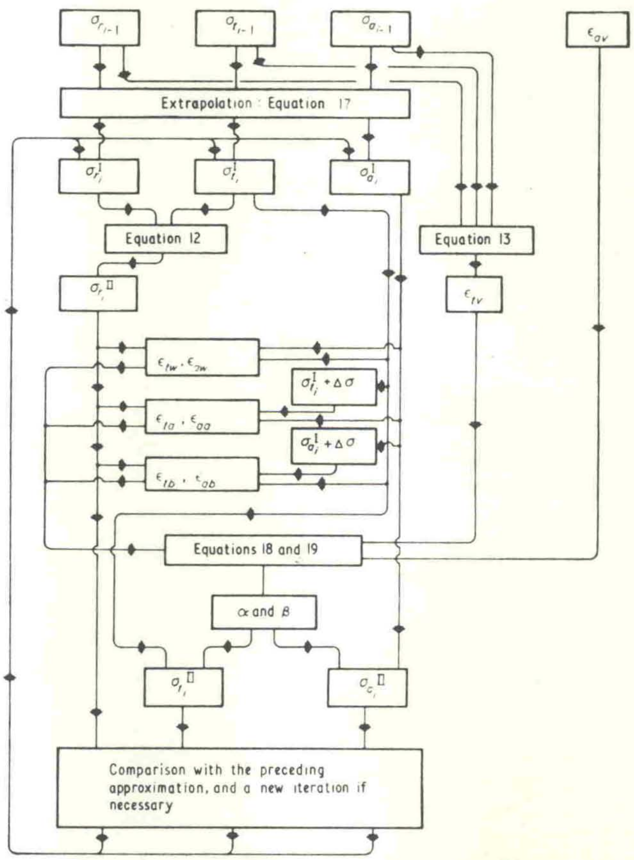

Fig. 9. Layout of a complete iteration